Federal Board HSSC-II Examination
Model Question Paper Mathematics
(Curriculum 2022-2023)

ROLL NUMBER					
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
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Version No.			
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1	1	1	1
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Section - A (Marks 20)

Time Allowed: 25 minutes

Section – A is compulsory.
All parts of this section are to be answered on this page and handed over to the Centre Superintendent.
Deleting/overwriting is not allowed.
Do not use lead pencil.

Invigilator Sign. ______________ **Candidate Sign.** ________________

Q1. Fill the relevant bubble against each question. Each part carries ONE mark.

Sr No.	Question	A	B	C	D	A	B	C	D
i.	The inverse of $f(x)=\frac{x-1}{2}$ is:	$2x-1$	$2x+1$	$x-2$	$x+2$	O	O	O	O
ii.	For what value of x, $f(x)=\frac{x-2}{x^2-4}$ is discontinuous?	-2	-1	1	0	O	O	O	O
iii.	What is the derivative of $\left(\frac{\sin\alpha}{\sec x}\right)$ w. r. t. x?	$\sin\alpha\ \sin x$	$-\cos\alpha\ \cos x$	$-\sin\alpha\ \sin x$	$-\cos x\ \sin\alpha$	O	O	O	O
iv.	If $y=\sin x$, then what is the value of y_4?	$sinx$	$cosx$	$-sinx$	$-cosx$	O	O	O	O
v.	The absolute maximum value of $f(x)=-x^2+3x-2$ over $[1,3]$ is:	$\frac{3}{2}$	$\frac{1}{4}$	3	-2	O	O	O	O
vi.	What is the integrated value of $(3x+1)^5$ w.r.t. x?	$\frac{(3x+1)^6}{18}+C$	$\frac{(3x+1)^6}{6}+C$	$\frac{3(3x+1)^6}{6}+C$	$\frac{\left(\frac{3x^2}{2}+x\right)^6}{2}+C$	O	O	O	O
vii.	What evaluates $\int_{-a}^{a}(x^3+x)dx$?	$2\int_0^a(x^3+x)dx$	0	$\int_{-a}^{0}(x^3+x)dx - \int_{0}^{a}(x^3+x)dx$	$\int_0^a(x^3+x)dx$	O	O	O	O
viii.	If $\int_2^4 f(x)\,dx=6$, then what results $\int_2^4[f(x)+3]\,dx$?	12	3	6	9	O	O	O	O

ix.	A car accelerates uniformly from rest and travels a distance of 100 m in 10 s. What is the acceleration of the car?	$1m/s^2$	$2m/s^2$	$4m/s^2$	$0.5m/s^2$	O O O O
x.	In which real-life scenario would the concept of first order differential equations be most applicable?	Predicting the speed of a moving car	Modeling the cooling of a hot beverage	Designing an electric circuit	Calculating the area of a triangle	O O O O
xi.	What is the next approximation, correct to two decimal places, for $f(x)=x^3-x^2+4x-4=0$ with an initial guess $x=2$ using the Newton-Raphson Method?	0.67	1.00	1.33	1.50	O O O O
xii.	If $a^2+b^2-c^2+2ab=0$, then family of straight lines $ax+by+c=0$ is concurrent at the points:	$(1,-1)$	$(-1,1)$	$(1,-2)$	$(1,1)$	O O O O
xiii.	The slope of a line perpendicular to a vertical line is:	0	1	-1	undefined	O O O O
xiv.	If $\vec{r}(t)=\ln(t-2)\hat{i}+\sqrt{4-t}\,\hat{j}$ $+e^t\,\hat{k}$, then domain of $\vec{r}(t)$ is:	$t>2$	$2<t\le 4$	$t\ge 2$	$t\in\mathcal{R}$	O O O O
xv.	If $\vec{V}(t)=\cos(t)\hat{i}+2\sin(2t)\hat{j}$ $+3t\hat{k}$, then acceleration $\vec{a}$ in the same direction is:	$-\sin(t)\hat{i}+$ $2\cos(2t)\hat{j}+3$	$-\sin(t)\hat{i}+$ $4\cos(2t)\hat{j}+3t\hat{k}$	$-\sin(t)\hat{i}+$ $2\cos(2t)\hat{j}+3\hat{k}$	$-\sin(t)\hat{i}+$ $4\cos(2t)\hat{j}+3\hat{k}$	O O O O
xvi.	What is the principal value of $Cot^{-1}\left(-\sqrt{3}\right)$?	$-\frac{\pi}{6}$	$\frac{5\pi}{6}$	$-\frac{\pi}{3}$	$\frac{2\pi}{3}$	O O O O
xvii.	What is the general solution of the trigonometric equation $sin(x)=\frac{\sqrt{3}}{2}$?	$-\frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$-\frac{\pi}{3}$	O O O O
xviii.	If center of the circle $x^2+y^2+mx+ny+2=0$ lies at $(4,-8)$, then what is the value of $m+n$?	-8	-4	8	4	O O O O
xix.	For what value of a, line $2x-3y+6=0$ is tangent to the parabola $y^2=4ax$?	$-\frac{4}{3}$	$-\frac{3}{4}$	$\frac{4}{3}$	$\frac{3}{4}$	O O O O
xx.	If $x-y+\lambda=0$ is tangent to ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, then value of λ is:	$\pm\sqrt{13}$	± 2	$\pm\sqrt{5}$	± 3	O O O O

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Time allowed: 2.35 hours | Total Marks: 80

Note: Answer all parts from Section 'B' and all questions from Section 'C' on the E-sheet. Write your answers on the allotted/given spaces.

SECTION – B (Marks 48)

$(12 \times 4 = 48)$

Q.2	Question	Marks		Question	Marks
i.	Draw the graph of the function $log_3 y = x$	4	***OR***	Solve the separable differential equation: $\frac{dy}{dx} = \frac{y^2 - 1}{x}$ Determine the general solution and express y in terms of x.	4
ii.	Evaluate: $\lim_{x \to 0} \left(\frac{\frac{1}{x-1} + \frac{1}{x+1}}{4x} \right)$	4	***OR***	Use the bisection method to approximate a root of the nonlinear equation: $f(x) = x^3 - x - 2$ in the interval $[1, 2]$. Perform three iterations and find the following: a. The midpoint at each iteration. b. The value of $f(x)$ at the midpoint. c. The approximate root after three iterations.	4
iii.	Test the continuity of the function $f(x) = \begin{cases} 1-3x, & x < -3 \\ 5, & x = -3 \\ x^2, & x > -3 \end{cases}$ at $x = -3$	4	***OR***	A surveillance drone is flying along a straight path defined by the equation $3x + 4y - 12 = 0$, while a stationary communication tower is located at (2,3). a. Find the shortest distance between the drone's path and the communication tower. b. Interpret the result in the context of the real-world scenario.	4
iv.	If $y = x^3 \ln\left(\sqrt{x^2+1}\right)$, then find $\frac{dy}{dx}$.	4	***OR***	The second-degree homogeneous equation $4x^2 + 6xy + y^2 = 0$ represents a pair of straight lines passing through origin. Find an acute angle θ between the two lines.	4
v.	Use the second derivative test to determine whether each critical point corresponds to relative maxima, minima or neither, if $f(x) = 3x^5 - 5x^3 + 2$.	4	***OR***	If $\vec{r}(t) = e^{2t}\,\hat{i} + e^{-t}\,\hat{j} + e^{t}\,\hat{k}$ at $t = ln2$ is the position vector of a particle, then find its velocity and acceleration at the given value of t in the direction of motion.	4
vi.	Find the derivative of the function w.r.t.x: $f(x) = \mathrm{Sin}^{-1}\left(\frac{x}{2}\right) + \mathrm{Tan}^{-1}\left(\frac{\sqrt{4-x^2}}{x}\right)$.	4	***OR***	A drone is programmed to follow the path given by the position vector-valued function: $\vec{r}(t) = 5t\,\hat{i} + 3\sin(t)\,\hat{j} + 2\cos(t)\,\hat{k}$, $0 \le t \le 2\pi$ a. Determine the maximum distance the drone travels in the y-direction. b. At what time does the drone return to the $z = 0$ plane?	4
vii.	A stone is thrown in the air. Its height at any time t is given by $h = -5t^2 + 10t + 4$. Apply derivatives to find the maximum height of	4	***OR***	A hyperbola of eccentricity $\frac{5}{2}$ has one focus at $(1, -4)$. The corresponding	4

	the stone.			directrix is the line $y=2$. Find equation of the hyperbola.	
viii.	Find the area of the region enclosed between the curves $y=4-x^2$ and $y=x^2$.	4	***OR***	Draw the graph of $y=Cos^{-1}\left(\frac{1-x}{3}\right)$ where $x\in[0,2]$.	4
ix.	The region bounded by $y=x^2$ and $y=x+2$ is revolved around *x*-axis. Calculate the volume of a solid formed by the revolution of this region.	4	***OR***	Solve the trigonometric equation $2cos^2(x)-3cos(x)+1=0$ in the interval $[0,2\pi]$.	4
x.	A car starts from rest and accelerates along a straight road. The acceleration of the car is given by $a(t)=3t\, m/s^2,\, t\geq 0$ where t is time in seconds. a. Find the velocity $v(t)$ of the car at any time t, given that the initial velocity $v(0)=0$. b. Determine the displacement $s(t)$ of the car at any time t, given that the initial displacement $s(0)=0$.	4	***OR***	Find the equations of tangent and normal to the parabola $x^2=8y$ at a point $(4,2)$.	4
xi.	Estimate the value of the integral $I=\int_0^{\pi} sin(x)dx$, using the trapezium rule with 4 subintervals. Round your answer to 4 decimal places.	4	***OR***	Find the equation of a circle passing through two points $(2,6),(6,4)$ with its centre on the line $3x+2y-1=0$.	4
xii.	Evaluate: $\int \frac{dx}{x^2\sqrt{4-x^2}}$.	4	***OR***	Without using calculator, prove that $Csc^{-1}\left(\frac{2\sqrt{3}}{3}\right)-Csc^{-1}\left(\frac{2}{\sqrt{2}}\right)=\frac{\pi}{12}$	4

SECTION – C (Marks 32)

$(4\times 8=32)$

Note: Attempt all questions. Marks of each question are given.

Q. No.	Question	Marks		Question	Marks
Q3	$y=3e^{2x}+2e^{3x}$, then prove that $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$	8	***OR***	Evaluate: $\int \frac{x^2+x-2}{3x^3-x^2+3x-1}dx$	8
Q4	Given the vertices of a triangle A (2,3), B (8,7), and C (4,11): a. Find the equation of the altitude from vertex A. b. Find the equation of the right bisector of side BC. c. Verify that the altitude and the right bisector intersect on the circumcircle of the triangle.	8	***OR***	An engineer is designing a bridge over a river. The bridge will have an inclined ramp that forms an angle of elevation with the ground. The total height of the bridge at its peak is 20 meters, and the base of the ramp is 50 meters long. a. Determine the angle of elevation of the ramp using an appropriate inverse trigonometric function. b. If the engineer wants to increase the height of the bridge to 30 meters while keeping the base length constant, calculate the new angle of elevation.	8
Q5	Solve the first-order homogeneous differential equation: $(x^2+y^2)dx=2xydy$	8	***OR***	A satellite orbits Earth in an elliptic trajectory. The semi major axis of the ellipse is10000 km, and the semi minor axis is 8000 km. Find a. The eccentricity of the orbit	8

	Also, determine the particular solution that passes through the point (1,2).			b. The distance of the satellite from earth's centre at the closest point if earth is at one focus. c. The distance of the satellite from earth's centre at the farthest point.	
Q6	A runner accelerates uniformly from rest to a velocity of 8 m/s in 4 seconds, maintains this velocity for 6 seconds, and then decelerates uniformly to rest in 5 seconds. a. Sketch the velocity-time graph for this motion. b. Interpret the graph to determine: (i) The total time of motion. (ii) The total distance covered by the runner.	8	***OR***	A company's stock price $S(t)$ (in dollars) is modeled by the function: $S(t)=50+20e^{-0.1t}$, where t is the time in years; a. Find the initial stock price. b. Determine the stock price after 5 years. c. Find the limit of $S(t)$ as $t\to\infty$. d. Determine the time at which the stock price reaches 60 dollars.	8